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Marangoni–Bénard convective instability driven by a heated divider

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Abstract—In the absence of gravitational effects, threshold values for the onset of convective thermocapillary instability are provided for a 'floating' liquid layer open to the ambient air and subjected to internal heating along a divider located in a plane midway between two open boundaries. For steady modes of instability both the critical Marangoni number and the critical wavenumber are given as functions of the pervious and thermal properties of the heated divider. Also provided here are numerical estimates and velocity profiles amenable to experimental test under *microgravity* conditions.

1. INTRODUCTION

A PARADIGMATIC case of surface tension-driven convection is the Bénard instability [1–5] where steady cellular convective motions arise in a horizontal liquid layer heated from a solid bottom. Indeed, although buoyancy and thermocapillary effects can be invoked to account for the instability the major cause is the Marangoni effect [6, 7], i.e. the surface stresses due to the temperature induced non-uniformity of the surface tension along the open boundary [8, 9]. In a space-craft, however, the *effective* gravity may be drastically reduced thus leading to the relatively higher importance of interfacial phenomena over buoyancy. Hence the expected possibility of containerless processing of materials in space demands studying all possible Marangoni-driven flows [10]. In the present paper we discuss a particular case of such steady instability as a result of internal heating along a solid 'divider' located in the plane midway between the two open boundaries of a liquid layer 'floating' in space. The 'divider' is assumed to be a thin pervious 'partition' separating the liquid layer in two halves. It could be a pervious copper divider, with suitable porosity, which may very well serve a grid catalyst for possible exo- or endothermic chemical reactions. When convective instability develops, whether or not both halves behave as two separate Marangoni–Bénard layers crucially depends on the properties of the 'divider'. In Section 2 we state the problem and make precise the role of both the permeability and the thermal conductivity of the 'divider'. In Section 3 we provide threshold conditions for instability and velocity profiles in terms of such properties of the 'divider'. Our results, amenable to experimental test under low/microgravity conditions, provide knowledge for the potential use of 'dividers' in controling flows in space.

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2. FORMULATION OF THE PROBLEM

Let us consider a 'floating' liquid layer ($h < z < -h$) with two outer free level surfaces open to air (considered passive), at $z = h$ and $z = -h$, respectively. At $z = 0$, i.e. midway between the two open boundaries we insert a pervious 'divider', thickness $2d$ ($d \ll h$), taken as a 'partition' which on the average is uniformly heated. A similar problem in buoyancy-driven Rayleigh–Bénard convection has already been considered by earlier authors, both theoretically [11, 12] and experimentally [13, 14]. Disregarding surface deformability and buoyancy in the liquid layer, here we shall consider the result of the

NOMENCLATURE

d half thickness of the divider
h half liquid layer depth
k horizontal Fourier mode
M Marangoni number, $\gamma\Theta h/\eta\chi$
p pressure
T temperature
$\mathbf{v}$ velocity vector
v_x, v_z horizontal and vertical velocity components
v dimensionless amplitude of the vertical velocity component
x, z horizontal and vertical coordinates.

Greek symbols
$\alpha, \alpha_\tau, \alpha_n$ $(\tilde{\alpha}_\tau, \tilde{\alpha}_n)$ generic 'divider'/partition resistance, tangential/parallel and transverse/normal resistances (quantities with tilde are dimensionless)
γ surface tension temperature coefficient, $-\partial\sigma/\partial T$
Δ Laplacian operator
η dynamic viscosity
θ, Θ dimensionless temperature disturbance and dimensional reference temperature
κ (inverse) Biot number
κ_f, κ_p liquid and 'divider'/partition heat conductivities
ν kinematic viscosity
σ surface tension
χ liquid heat diffusivity.

Marangoni effect acting at two opposite surface boundaries.

In the quiescent state the temperature profile is assumed to be initially linear

$$T_0 = \Theta \times \begin{cases} (1-z/h) & \text{at} \quad z \geqslant 0 \\ (1+z/h) & \text{at} \quad z < 0 \end{cases} \tag{1}$$

with Θ given. Let $(T, p, \mathbf{v})$ denote infinitesimal temperature, pressure and velocity disturbances. They obey the linearized form of the Navier–Stokes, continuity and heat transport equations:

$$\frac{\partial \mathbf{v}}{\partial t} = -\frac{1}{\rho}\nabla p + \nu\Delta\mathbf{v} \tag{2a}$$

$$\operatorname{div} \mathbf{v} = 0 \tag{2b}$$

$$\frac{\partial T}{\partial t} + \mathbf{v}\cdot\nabla T_0 = \chi\Delta T \tag{2c}$$

where ρ is the density, ν is the *kinematic* viscosity and χ the heat diffusivity of the liquid.

At the open boundaries we assume

$$v_z|_{z=\pm h} = 0 \tag{3}$$

together with the tangential stress balance. If the surface tension of the open boundaries depends linearly on the temperature

$$\sigma = \sigma_0 - \gamma T \tag{4}$$

with γ generally taken positive, then the boundary condition (b.c.) is

$$z = \pm h: \ \eta\frac{\partial v_x}{\partial z} = \mp\gamma\frac{\partial T}{\partial x} \tag{5}$$

where η is the *dynamic* viscosity. The disturbed heat flux at both free surfaces is assumed such that:

$$\left.\frac{\partial T}{\partial z}\right|_{z=\pm h} = 0. \tag{6}$$

At the 'partition', of negligible thickness with respect to the depth of the liquid, we assume continuity of the velocity, with through-flow obeying Stokes law, i.e. the tangential velocity component is taken proportional to the sum of the tangential stresses on both sides of the partition while the normal component is taken proportional to the pressure jump across it [15]. Thus we have at $z = 0$:

$$v_x^+ = v_x^-, \quad v_z^+ = v_z^- \tag{7a}$$

$$v_x = \alpha_\tau^{-1}(\eta\,\partial v_x^+/\partial z - \eta\,\partial v_x^-/\partial z) \quad \text{and}$$

$$v_z = -\alpha_n^{-1}(p^+ - p^-) \tag{7b}$$

where α_τ and α_n are phenomenological parameters that define the characteristic tangential/parallel and normal/transverse hydrodynamic 'resistances' of the 'divider'. Thermally, this 'partition' is such that [16]

$$T^+ = T^- \tag{8a}$$

$$-\kappa_f(\partial T^+/\partial z - \partial T^-/\partial z) = \kappa_p 2d\Delta_1 T \tag{8b}$$

with + and − denoting just above and below it, respectively; $\Delta_1 = (\partial^2/\partial x^2 + \partial^2/\partial y^2)$, κ_f and κ_p the heat conductivity of the fluid and the 'divider', respectively.

Now we introduce dimensionless variables using suitable *units*; distance: h; time: h^2/ν; velocity: χ/h; partition 'resistance': η/h and temperature: Θ. Then with these new 'scales' we obtain the nondimensional form of the equations and boundary conditions. Using the Fourier decomposition $v_z = v(z)\,e^{ikx}$ and $T = \theta(z)\,e^{ikx}$, with primes (′) denoting z-derivatives and restricting consideration to steady modes of instability we have:

$$v^{iv} - 2k^2 v'' + k^4 v = 0 \tag{9a}$$

$$\theta'' - k^2\theta \pm v = 0 \tag{9b}$$

with b.c.

- at $z = 0$:

$$[v^+ = v^-, v^{+\prime} = v^{-\prime}; \quad v^{+\prime\prime\prime} - v^{-\prime\prime\prime} = -k^2\alpha_n v;$$

$$v^{+\prime\prime} - v^{-\prime\prime} = \alpha_\tau v'; \quad \theta^+ = \theta^-; \quad \theta^{+\prime} - \theta^{-\prime} = 2k^2\kappa\theta] \tag{10a}$$

and

- at $z = \pm 1$:

$$[v = 0, -v'' = \pm Mk^2\theta, \theta' = 0] \tag{10b}$$

with $\alpha_n = \tilde{\alpha}_n h/\eta$, $\alpha_\tau = \tilde{\alpha}_\tau h/\eta$ (quantities with tilde are dimensionless), $\kappa = d\kappa_p/h\kappa_f$ (inverse) Biot number, and $M = \gamma\Theta h/\eta\chi$, Marangoni number.

The solutions of equations (9) and (10) naturally separate into *even* and *odd* modes:

(i) For the *even* solution: $v(-z) = v(z)$, $\theta(-z) = -\theta(z)$, with b.c.

- at $z = 0^+$:

$$[v' = 0, v''' = -k^2\tilde{\alpha}_n v/2, \theta = 0],$$

and

- $z = +1$:

$$[v = 0, v'' = -Mk^2\theta, \theta' = 0].$$

(ii) For the *odd* solution: $v(-z) = -v(z)$, $\theta(-z) = \theta(z)$, with b.c.

- at $z = 0^+$:

$$[v = 0, v'' = \alpha_\tau v'/2, \theta' = k^2\kappa\theta],$$

and

- at $z = +1$:

$$[v = 0, v'' = -Mk^2\theta, \theta' = 0].$$

We clearly see that the *even* solution depends on *one* parameter only, namely $\tilde{\alpha}_n$, while the *odd* mode depends on *two* parameters, $\tilde{\alpha}_\tau$ and κ.

The general solution of both problems can be written in the region $z > 0$ as

$$v = c_1 \operatorname{sh} kz + c_2 \operatorname{ch} kz + c_3 z \operatorname{ch} kz + c_4 z \operatorname{sh} kz \tag{11}$$

and

$$\theta = c_5 \operatorname{sh} kz + c_6 \operatorname{ch} kz + (c_4/4k^2 - c_2/2k) z \operatorname{sh} kz + (c_3/4k^2 - c_1/2k) z \operatorname{ch} kz - (c_3 \operatorname{sh} kz + c_4 \operatorname{ch} kz) z^2/4k \tag{12}$$

where the c_i ($i = 1, \ldots, 6$) are constants to be fixed by the b.c.

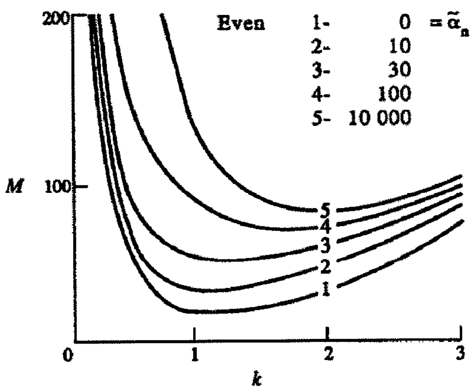

FIG. 1. Neutral stability curves (*even* solution) at fixed values of the transverse 'resistance' of the divider, $\tilde{\alpha}_n$.

3. THRESHOLD CONDITIONS FOR INSTABILITY AND EXPECTED VELOCITY PROFILES

The problem is now reduced to solving a determinantal equation that upon separation of the solution in its *even* and *odd* parts yields:

(i) *even* modes

$$M = [32k^2 + 8\tilde{\alpha}_n(k^2t^2 + kt - k^2)]/[4(k^2 - 2kt + t^2 - k^2t^2 + 2kt^3) + \tilde{\alpha}_n(t^3/k - k^2 + k^2t^2)] \tag{13}$$

with $t = \tanh k$. Asymptotically we have:

(i.a) $k \to 0$:

$$M = [480 + 80\tilde{\alpha}_n k^2]/[60k^2 + \tilde{\alpha}_n k^4] \tag{14a}$$

(i.b) $k \to \infty$:

$$M \approx 8k^2. \tag{14b}$$

Thus the threshold values correspond to the minima of the curves shown in Fig. 1. At $\tilde{\alpha}_n = 0$ we have $M_c = 21.8$ and $k_c = 1.1$, while at $\tilde{\alpha}_n = 10^4$ (practical 'infinity') we have $M_c = 79.5$ and $k_c = 2.0$. The latter results for an impervious partition reproduce the findings earlier known from ref. [2] for a *good conducting* heater. Figure 2 provides the vertical velocity in the upper half layer and related flow profiles as $\tilde{\alpha}_n$ increases from zero to (practical) 'infinity'. Very much like in the case of buoyancy-driven Rayleigh–Bénard convection [11] we see the crucial role played by $\tilde{\alpha}_n$ in establishing a *single cell* convective pattern all over the two halves of the liquid layer. Indeed, for the *even* solution when $\tilde{\alpha}_n$ tends to zero the 'divider' plays little mechanical role. In the opposite case when $\tilde{\alpha}_n$ tends to infinity the 'partition' truly divides the liquid into two separate Marangoni–Bénard layers. For completeness, Fig. 3 depicts the expected vertical velocity component at vanishing $\tilde{\alpha}_n$ for various values of the wavenumber at neutral stability conditions.

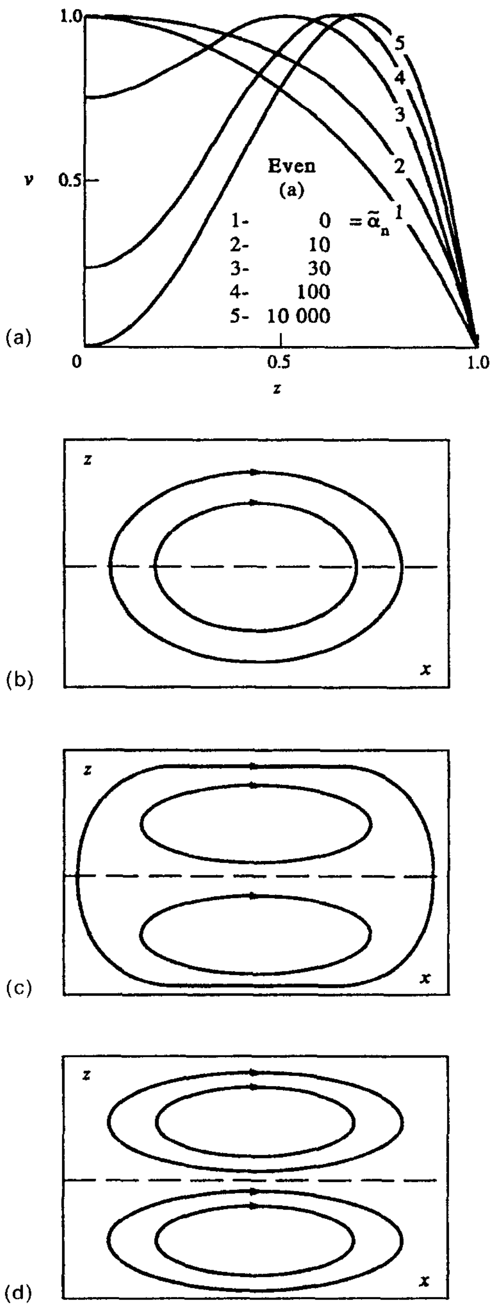

FIG. 2. *Even* solution: (a) vertical velocity component of the expected flow at threshold for several values of the transverse 'resistance' of the divider α_n, and qualitative sketch of the corresponding flow profiles: (b) for $\tilde{\alpha}_n = 0$, (c) for $\tilde{\alpha}_n = 30$, and (d) for $\tilde{\alpha}_n = 10^4$. Note that at large enough values of the 'resistance' the divider strictly splits the layer in two halves.

(ii) *odd* modes

$$M = [16k^2t^3+4\tilde{\alpha}_\tau(k^2t^3+kt^2-k^2t)+16\kappa k^3t^2 \\ +4\tilde{\alpha}_\tau\kappa(k^3t^2+k^2t-k^3)]/[2(t^3-k^2t+k^2t^3) \\ +0.5\tilde{\alpha}_\tau(k+t^2/k-2t+2t^3-k^2t+k^2t^3-kt^2) \\ +2\kappa(2k^3t^2-k^2t^3+k^2t-2k^3+kt^2) \\ +0.5\tilde{\alpha}_\tau\kappa(t^3+k^3t^2-k^3)]. \qquad (15)$$

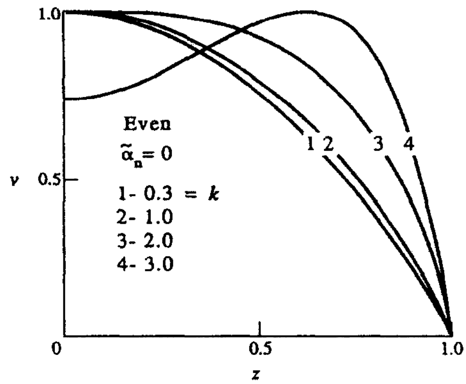

FIG. 3. *Even* solution: vertical velocity component of *neutral* disturbances for vanishing $\tilde{\alpha}_n$. Values of k correspond to critical values at threshold.

Asymptotically we have:

(ii.a) $k = 0$:

$$M = [288+48\tilde{\alpha}_\tau+288\kappa+48\tilde{\alpha}_\tau\kappa]/[12+\tilde{\alpha}_\tau] \qquad (16a)$$

(ii.b) $k \to \infty$:

$$M \approx [16k^2+4\tilde{\alpha}_\tau k+16\kappa k^3+4\tilde{\alpha}_\tau\kappa k^2]/ \\ [2+0.5\tilde{\alpha}_\tau/k+2\kappa k+0.5\tilde{\alpha}_\tau\kappa]. \qquad (16b)$$

The threshold values correspond to the minima of the curves shown in Fig. 4, which, for illustration, correspond to $\kappa = 0$, $\kappa = 1$ and $\kappa = 5$, respectively. For $\kappa = 0$, at $\tilde{\alpha}_\tau = 0$ we have $M_c = 24$ and $k_c = 0$, while at $\tilde{\alpha}_\tau = 10^4$ (practical 'infinity') we have $M_c = 48$ and $k_c = 0$. The latter results for a *poor rigid conducting* 'divider' reproduce the findings also known from ref. [2]. When κ does not vanish, e.g. $\kappa = 5$ the threshold values do not correspond to a vanishing wave number and are: at $\tilde{\alpha}_\tau = 0$: $M_c = 54.8$ with $k_c = 1.6$ and at $\tilde{\alpha}_\tau = 10^4$ (practical 'infinity'): $M_c = 77.8$ with $k_c = 1.9$. Figure 5 depicts the predicted vertical velocity component for the *odd*

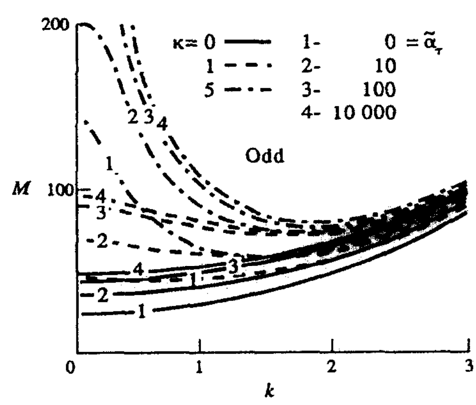

FIG. 4. Neutral stability curves (*odd* solution) at fixed values of the parallel 'resistance' of the divider, $\tilde{\alpha}_\tau$, and different values of the (inverse) Biot number, κ.

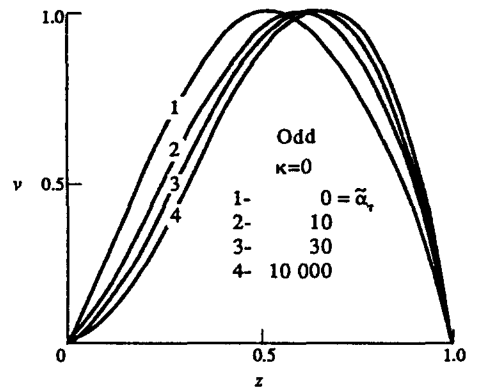

FIG. 5. *Odd* solution: vertical component of the expected flow at threshold for several values of the parallel 'resistance' of the divider, $\tilde{\alpha}_\tau$, and vanishing heat conductivity.

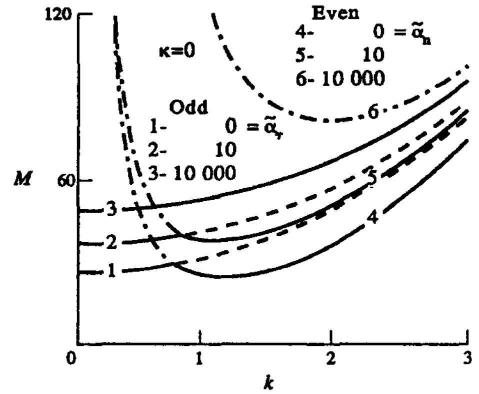

FIG. 6. Neutral curves (*even* and *odd* disturbances) at vanishing κ. Solid line portions denote the lower threshold, most dangerous instability mode at equal transverse and parallel 'resistances', $\tilde{\alpha}_n = \tilde{\alpha}_\tau$.

solution. It clearly shows that at variance with the *even* mode for the *odd* mode both halves develop separate convection cells as it may intuitively be guessed. On the other hand for the latter case it appears that the flow pattern shows only a slight dependence on the (inverse) Biot number κ. Both *even* and *odd* convective modes show velocity profiles strongly dependent on the corresponding 'resistance'. For small values of $\tilde{\alpha}_n$, i.e. in the case of a good 'permeability' of the 'divider' we see that the two initially unrelated Marangoni-driven instabilities yield flows that merge the two expected steady cell patterns into a single convective structure. In the opposite extreme case, a rigid partition merely yields the usual Marangoni–Bénard convection [2] in the two halves.

4. CONCLUSION

The *steady* Marangoni–Bénard convective instability of a 'floating' liquid layer heated at a pervious to through-flow divider located midway between its two open surfaces has been considered. We have shown how, in the absence of gravity, the threshold for instability drastically depends on the mechanical characteristics of this partition as well as on its heat transfer properties. The solution of the boundary value problem for *neutral* disturbances naturally splits into *even* and *odd* modes. For the *even* solution there is a critical Marangoni number that depends only on the partition's transverse hydrodynamic 'resistance', $\tilde{\alpha}_n$. For the *odd* solution the critical Marangoni number depends on two parameters, the tangential 'resistance' of the divider, $\tilde{\alpha}_\tau$, and its (inverse) Biot number κ.

Comparison between the expected behavior of *even* and *odd* modes of convective instability is illustrated in Fig. 6. For the particular case of vanishing (inverse) Biot number, $\kappa = 0$, it appears that in the long wave region *odd* disturbances have a lower threshold while in the short wave region, for small 'resistance' values, the *even* modes are more dangerous. Note that for *even* modes, under appropriate conditions the two halves of the layer behave like a single convective cell while otherwise both halves may convect in a rather unrelated way. Overall, for large enough values of the 'resistances', $\tilde{\alpha}_n$ and $\tilde{\alpha}_\tau$, the *odd* modes are the most dangerous. From the behavior of all *neutral* stability curves at vanishing wavelengths it can be shown that all critical Marangoni numbers, M_c, asymptote to $8k^2$.

Figure 7 depicts the dependence of the critical Marangoni number, M_c, with the corresponding hydrodynamic 'resistance' of the 'divider'/partition, α: for *even* modes—curve #1—$\alpha \equiv \tilde{\alpha}_\tau$, and for *odd* modes—curves #2–6—$\alpha \equiv \tilde{\alpha}_n$. There is a rapid variation of M_c in the region $1 < \alpha < 100$. The measurement of the hydrodynamical 'resistances' of the partition [14] shows that the upper value of this interval fits well with a 'divider' with not so 'small' holes. Even though such partition suppresses through-flow yet it allows for molecular diffusion from one half to the other in the liquid layer.

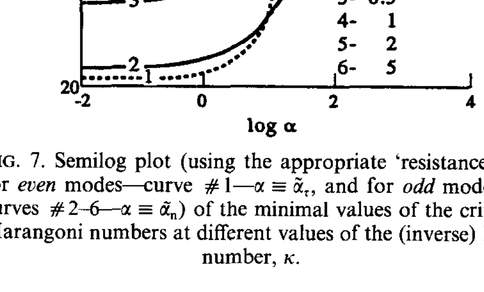

FIG. 7. Semilog plot (using the appropriate 'resistance' α: for *even* modes—curve #1—$\alpha \equiv \tilde{\alpha}_\tau$, and for *odd* modes—curves #2–6—$\alpha \equiv \tilde{\alpha}_n$) of the minimal values of the critical Marangoni numbers at different values of the (inverse) Biot number, κ.

In conclusion, we can say that in view of the drastic influence of the partition on the expected Marangoni–Bénard flows, the use of a heated pervious 'divider', e.g. as a grid catalyst or some other device, shows potential usefulness to control flows in *microgravity* conditions.

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